A NUMERICAL METHOD FOR THE SOLUTION OF PLATE BUCKLING PROBLEMS

BY

WALTER JAMES AUSTIN

B.S., The Rice Institute, 1941
M.S., University of Illinois, 1946

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
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Introduction

The problems of plate buckling in the elastic range are of importance in many fields where assemblies of plates and stringers are used. The problem was first investigated in response to a demand for information by ship builders. At the present time interest largely centers around aircraft construction. In aircraft many sections are so light that elastic stability is the controlling factor. Some plates in aircraft structures are designed to work at loads exceeding the so-called critical buckling load. The designer depends on the increase in strength which the plate gains when the deflections grow large. Hence the problem of determining the buckling behavior in the large deflection range is of interest.

The basic theory of buckling in the small deflection range was well established by the latter part of the last century. Yet up to the present time only a few analytical solutions exist, and these for simple boundary conditions. The solution of the partial differential equation which describes the action of buckling is mathematically difficult. Most analytical solutions are semi-exact, using some type of variational technique.

The basic theory of large deflections has been established since the early part of this century. The problem of buckling in the large deflection range is much more difficult than that for the small deflection range because two, simultaneous, nonlinear, partial differential equations must be solved. Up to the present time only a very few solutions exist.

The object of this thesis is to develop a numerical method of solving such plate buckling problems. Numerical methods have the advantage that they are not mathematically complex and they are versatile since almost any problem conceived can be solved in a straightforward manner. The greatest emphasis

has been placed on the problem of buckling in the small deflection range.

Discussion Of Methods

The numerical method developed is based on substituting finite difference approximations for derivatives in the pertinent differential equations. As usual, the plate is divided into a network of squares; the intersections of these network lines are called node points. The buckled shape of the plate is described by the deflections at the node points of the gridwork. At each node point the differential equation is expressed in its finite difference form. This results in a system of linear, homogeneous, simultaneous equations relating the deflections at the various node points. These equations are solved by a special relaxation-iteration procedure. Several schemes to speed up the convergence are developed so that the labor has been greatly reduced. The method is rapid and simple.

A special method has been developed for problems of buckling of rectangular plates where the deflection function can be assumed to be the product of a sine wave in one direction times an unknown function in the transverse direction. In this case the problem reduces to that of finding a function of one variable. The solution for this case can be made very rapidly. It is interesting to note that a great many of the analytical solutions fall in this class.

The only problems explicitly solved in this thesis are those of rectangular plates with hinged or fixed edges. Irregular boundaries can be treated numerically, but additional study needs to be made of this point.

The method is extended to treat the problem of the buckling strength of ribbed plates. The only problems treated herein are those in which it is possible to arrange the calculation network on the plate so that the ribs lie on network lines. The strength of a plate of varying thickness can be easily treated by the method although no actual explanation is given for this problem.

The accuracy of a numerical solution depends on the fineness of the network used. As the network grows finer the values of the finite difference approximations approach the values of the equivalent derivatives. Therefore, the solution may be made as accurate as is desired by simply choosing an appropriate size for the network. Unfortunately, the use of a fine network increases the number of simultaneous equations and hence increases the amount of labor required for a solution. Also the finer the network the more significant figures that need to be used because the differences become smaller. For these reasons the coarsest possible network is generally used.

A method of extrapolating the correct value of the buckling load from two or three inaccurate solutions has been used successfully. By this procedure two or three simple solutions of the problem using coarse networks are made. Then by means of Richardson's evaluation of the errors inherent in finite difference expressions the correct answer is extrapolated. The method gives excellent results and it is much quicker to solve three problems with coarse networks than to solve one problem with a very fine network.

The solutions given in this thesis are for problems for which analytical solutions are available. In this way the accuracy and validity of the method are tested. No attempt has been made to work out solutions to practical problems for which answers are not available. This can be done by anyone interested.

The problem of elastic buckling in the large deflection range has been treated very briefly. When the deflections grow large (greater than about one-half the thickness of the plate) the membrane stresses induced by the deflection counteract to some extent the buckling stresses and so strengthen the plate. Therefore, the strength of a thin plate in the elastic state far exceeds the critical buckling load for small deflections. This increased strength is of practical interest whenever large, thin plates are used in compression. The theory for this action

was published by von Karman in 1910.

The action of a plate in buckling under large deflections has been described from several view points. Then a method of solving the two simultaneous, non-linear, differential equations which describe this action is given. Again the solution is accomplished by means of finite differences. Most of the procedures evolved for the problem of small deflections are used for this case also. Unfortunately the solution requires an enormous amount of work. However, since there are almost no mathematical solutions available numerical methods such as proposed here are useful. This thesis contains only two simple examples worked with very coarse networks. The solutions could not be checked with analytical solutions since there are none available for these problems.

The stress solutions are performed assuming that the stresses at the edge are known. No limitations are placed on displacements. However, in many practical problems the displacements are controlled in some fashion and the stresses are unknown. More study needs to be made on this aspect.

Conclusions

Problems of elastic buckling of plates in the small deflection range can be solved successfully by numerical methods such as proposed here. The technique is mathematically straightforward. Solutions with an accuracy sufficient for engineering purposes can be gained with a moderate amount of work. Highly accurate results can be gained at the expense of greater labor. The accuracy attainable can be judged from comparisons with exact solutions made for eighteen illustrative problems.

Problems of elastic buckling of plates in the large deflection range can be solved successfully by numerical methods but only at the cost of a great deal of labor. This problem has not been investigated extensively in this work. Further study needs to be made on it.

VITA

Walter J. Austin was born in St. Louis, Missouri on February 6, 1920. Upon graduation from Waller High School, Chicago, he entered the Rice Institute, Houston, Texas. He received the degree of Bachelor of Science in Civil Engineering in June, 1941.

He entered the University of Illinois in September, 1941, as a Special Research Graduate Assistant in Civil Engineering. During the school year he worked on the Concrete Slab Investigation assisting in the analysis, construction, and testing of a model skew-slab bridge.

In June, 1942, he accepted a position as structural draftsman and designer with the Chicago Bridge and Iron Company. He worked on many different types of structures there, including LST landing ships, floating drydocks, flat-bottom oil storage tanks, and elevated water tanks.

In March, 1946, he re-entered the University of Illinois as a Research Graduate Assistant. During 1946 he wrote a master's thesis entitled, "A Framework Analogy For Plane Problems in Elasticity". In September, 1946, he obtained the degree of Master of Science in Civil Engineering. He has continued his graduate studies since that time, for one year under a Fellowship and for the rest of the time as Research Associate, a position which he now holds.

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